Characterizing Temporal Distinguishability of an N-Photon State by Generalized Photon Bunching Effect with Multi-Photon Interference

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The complementary principle of quantum mechanics relates qualitatively the visibility of quantum interference with path indistinguishability. Here we propose a scheme of constructive quantum interference involving superposition between an N-photon state and a single-photon state to characterize quantitatively the degree of temporal distinguishability of the N-photon state. This scheme is based on a generalized photon bunching effect. Such a scheme can be extended to other more general cases.

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I. INTRODUCTION

The complementary principle of quantum mechanics was first proposed by Bohr [1] to deal with the wave-particle duality of quantum particles. On the one hand, it successfully explained the peculiar quantum behavior of particles in interference. On the other hand, it only provides a qualitative description of quantum interference process. The problem stems from the lack of a quantitative definition of distinguishability. Efforts were made to find such a definition with some success [2, 3, 4, 5].

The above mentioned discussions of the complementary principle were mostly confined in fundamental conceptual study and in interference involving only one particle. However, recent interests on quantum information involve quantum interference of multiple particles [6], especially in the context of linear optical quantum computing with qubits realized by photons[7]. An issue thus arises about distinguishability among the photons that may degrade the quantum interference effects, leading to poor performance of the quantum operations. So it is desirable to study photon distinguishability qualitatively and to find its relation with multi-photon quantum interference effect.

The first investigation of the effect of photon distinguishability on multi-photon interference was performed by Grice and Walmsley [8] with an analysis on a two-photon polarization Hong-Ou-Mandel interferometer [9]. A more complicated four-photon case was studied by Ou et al [10, 11] and later by Tsujino et al [12, 13] with concerns about the distinguishability between two pairs of photons.

Recently, the current author [14] made an attempt to generalize to the above discussion to a system of arbitrary number of photons. A degree of temporal distinguishability is quantitatively defined and a destructive multi-photon interference method is proposed that relies on a quantum state projection measurement [15, 16, 17] to experimentally measure it. Subsequent experimental demonstrations [18, 19] confirmed some of the predictions. It was shown [14] that the visibility of interference is proportional the number of indistinguishable photons in a simple situation. But since the visibility is bounded by 1, more accurate measurement on the visibility is required to distinct various scenarios of different photon distributions, especially when the photon number is large. The accuracy problem is compounded by the fact that destructive interference in this scheme makes the measured quantity small, so that it requires long recording time for good accuracy. Furthermore, the scheme of quantum state projection measurement is complicated in structure and requires phase shifters with precise values. Another scheme was recently discussed in Ref.[20] that relies on a generalized Hong-Ou-Mandel interference effect with asymmetric beam splitter. This scheme needs less optical elements and thus significantly simplifies the optical arrangement. But since this new scheme is based on destructive interference, it still suffers the problem of low count rate at maximum interference effect and thus low measurement accuracy. Another disadvantage in this new scheme is that we need to control the precise value of the transmissivity of the beam splitter, which depends on the total photon number, in order to achieve complete destructive multi-photon interference.

It was demonstrated that a photon pair bunching effect can be used to characterize temporal distinguishability between two pairs of photons [10, 11]. In this case, constructive four-photon interference for two pairs of photons leads to five-fold increase in four-photon coincidence when the two pairs are indistinguishable, whereas the state of two separated pairs produces only three-fold increase. The enhancement factor is expected to be bigger for larger photon number due to Bose statistics. Largeness of the measured quantity leads to a good accuracy in the four-photon coincidence measurement.

In this paper, we generalize the study of the photon bunching effect to arbitrary photon number. We find that the enhancement factor in photon bunching is due to constructive interference and can be used to characterize the temporal distinguishability of photons. The enhancement factor is not sensitive to the experimental parameters and the optical arrangement is relatively sim-

ple. This scheme seems to overcome all the shortcomings of previous schemes. The paper is arranged as follows: In Sect.II, we will discuss stimulated emission as a photon bunching effect due to constructive interference and exploit it for characterizing temporal distinguishability of incoming photons. We will also discuss its analogy with a beam splitter. This is a simple single mode analysis. In Sect.III, we will perform the more rigorous multi-mode analysis and confirm the results from the simple single mode analysis. In Sect.IV, we will consider other scenarios of photon temporal distributions and derive the corresponding enhancement factor. We end the paper with a discussion.

II. STIMULATED EMISSION AS A MULTI-PHOTON CONSTRUCTIVE INTERFERENCE EFFECT

Recently, it was pointed out [21] that stimulated emission can be interpreted as a result of multi-photon constructive interference: when N input photons are indistinguishable from the photon emitted by the excited atom, constructive interference leads to a factor of N enhancement in the atomic emission rate from spontaneous emission. The enhanced emission is due to stimulated emission. On the other hand, if the input photons are completely distinguishable from the photon emitted by the atom, no enhancement occurs and the atom makes only spontaneous emission.

If the input photons are partially indistinguishable from the emitted photon, only the indistinguishable part will give rise to the stimulated emission. Therefore, such a scheme can be used to characterize quantitatively the degree of distinguishability of the input photons. To see how this works, we consider an excited atom modelled as a phase insensitive quantum amplifier with small gain [22]:

$$\hat{a}_s^{(out)} = G\hat{a}_s + g\hat{a}_0^\dagger, \tag{1}$$

where $\hat{a}_0$ represents all the internal modes of the amplifier and it is usually independent of the signal mode $\hat{a}_s$ and is in vacuum. To preserve the commutation relation, we need $|G|^2 - |g|^2 = 1$ and for small gain, $|g| << 1$. The related evolution operator for the system has the form of

$$\hat{U} = \exp\{\eta\hat{a}_s^\dagger\hat{a}_0^\dagger - h.c.\} \approx 1 + (g\hat{a}_s^\dagger\hat{a}_0^\dagger + h.c.) \tag{2}$$

with $g \approx \eta$.

With a vacuum input of $|0\rangle$, we have the output state

$$|\Phi\rangle_{out}^{(0)} = \hat{U}|0\rangle \approx |0\rangle + g|1\rangle_s \otimes |1\rangle_0. \tag{3}$$

This gives the spontaneous emission probability of $|g|^2$. When the input is an N-photon state $|N\rangle_s \otimes |0\rangle_0$, we have

$$\begin{aligned} |\Phi\rangle_{out}^{(1)} &\approx |N\rangle_s|0\rangle_0 + g(\hat{a}_s^\dagger|N\rangle_s) \otimes (\hat{a}_0^\dagger|0\rangle_0) \\ &= |N\rangle_s|0\rangle_0 + g\sqrt{N+1}|N+1\rangle_s \otimes |1\rangle_0. \end{aligned} \tag{4}$$

The probability becomes $(N+1)|g|^2$. The stimulated emission helps to enhance the emission rate by a factor of $N+1$.

In Eq.(4), the input N photons are all in the same mode as the mode $\hat{a}_s$ of the amplifier. However, if some of the input photons are in different modes from the mode $\hat{a}_s$ of the amplifier, these photons are not coupled to the amplifier and cannot stimulate the emission of the amplifier. Mathematically, we have the input as $|m\rangle_s|N-m\rangle_{s'}|0\rangle_0$ and the output state as

$$\begin{aligned} |\Phi\rangle_{out}^{(1)'} &\approx |m\rangle_s|N-m\rangle_{s'}|0\rangle_0 \\ &\quad + g(\hat{a}_s^\dagger|m\rangle_s) \otimes (|N-m\rangle_{s'}) \otimes (\hat{a}_0^\dagger|0\rangle_0) \\ &= |m\rangle_s|N-m\rangle s'|0\rangle_0 \\ &\quad + g\sqrt{m+1}|m+1\rangle_s|N-m\rangle_{s'}|1\rangle_0. \end{aligned} \tag{5}$$

The enhancement factor is now $m+1$. In the special cases when $m = 0, N$, we recover Eqs.(3, 4), respectively. Therefore, spontaneous emission corresponds to the case when the input photons are completely distinguishable from the photon emitted from the amplifier whereas stimulated emission occurs when the input photons are indistinguishable from the photon emitted by the amplifier.

Notice that the enhancement factor $m+1$ is linearly related to the number of indistinguishable photons. Thus by observing the size of the enhancement, we can quantitatively characterize the degree of distinguishability. However, the mode of the amplifier is somewhat complicated, which makes this scheme hard to implement.

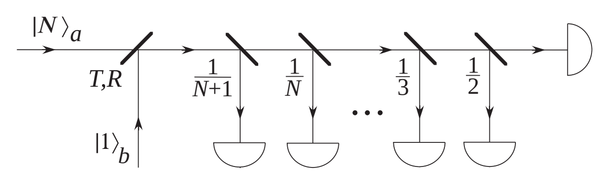

FIG. 1: Generalized photon bunching effect for characterizing the distinguishability of photons.

We can circumvent this problem with linear optics. As discussed before, the enhancement effect in stimulated emission is due to photon indistinguishability and is the result of constructive multi-photon interference, which can then be mimicked by a lossless beam splitter, as shown in Fig.1. Ref.[21] showed that the result in the scheme of Fig.1 for $N+1$ indistinguishable photons is the same as the stimulated emission process described by Eq.(4) with an enhancement factor of $N+1$ compared to the situation when the N photons are distinguishable from the single photon at the other side of the beam splitter.

For the case when $m \neq N$, i.e., the case when some of the input N photons are distinguishable, we may use a similar input state as in Eq.(5), i.e., $|N-m\rangle_{a'} \otimes |m\rangle_a|1\rangle_b$. In this state, the $N-m$ photons are distinguishable from the m photons and the single photon from the other side. Since the state $|m\rangle_a|1\rangle_b$ is the same as the result of stimulated emission with input state of $|m\rangle$ and the state

$|N-m\rangle_{a'}$ has no enhancement effect, the overall enhancement factor is simply $m+1$, exactly the same as the case of stimulated emission given in Eq.(5).

The enhancement effect with a beam splitter in Fig.1 is a generalized photon bunching effect and it can be similarly used for the characterizing the degree of photon distinguishability.

However, the above analysis is a single-mode analysis. To fully prove its validity, we need a multi-mode analysis.

III. MULTI-MODE ANALYSIS OF THE MULTI-PHOTON BUNCHING EFFECT

In this multi-mode analysis, we will only concentrate on the temporal/spectral mode and ignore other modes such as spatial and polarization modes. However, the generalization is straightforward.

Let us consider the scheme in Fig.1 where an N-photon state and a single-photon state enter a beam splitter from two separate sides (labelled as a and b). According to Ref.[14], an arbitrary $N+1$-photon state of a wide spectral range can be expressed for multi-mode analysis as

$$|\Phi_{N,1}\rangle = \mathcal{N}^{-1/2}\int d\omega_0 d\omega_1 d\omega_2 ... d\omega_N \Phi(\omega_0;\omega_1,...,\omega_N) \times \hat{b}^\dagger(\omega_0) a^\dagger(\omega_1)\hat{a}^\dagger(\omega_2)...\hat{a}^\dagger(\omega_N)|0\rangle, \quad (6)$$

where the normalization factor $\mathcal{N}$ is given by

$$\mathcal{N} = \int d\omega_0 d\omega_1 ... d\omega_N \Phi^*(\omega_0;\omega_1,...,\omega_N)\times \times \sum_{\mathbb{P}} \Phi(\omega_0;\mathbb{P}\{\omega_1,...,\omega_N\}) \quad (7)$$

with $\mathbb{P}$ as the permutation operator on the indices of 1, 2, ..., N. The sum is over all possible permutations. $\hat{a}$ and $\hat{b}$ represent the input modes a and b of the beam splitter, respectively.

Let us consider the situation when the single photon from input port b overlaps temporally with m photons from the N input photons at input port a and the rest of the $(N-m)$ photons in side a are completely distinguishable in time from the $(m+1)$ photons. From Ref.[14], we learn that the $N+1$-photon wave function satisfies the permutation symmetry relations:

$$\begin{aligned}\Phi(\omega_0;\omega_1,...,\omega_N) \\ = \Phi(\mathbb{P}\{\omega_0;\omega_1,...,\omega_m\},\omega_{m+1},...,\omega_N),\end{aligned} \quad (8)$$

for all permutation operation $\mathbb{P}$ and the orthogonal relations:

$$\int d\omega_0 d\omega_1 ... d\omega_N \Phi^*(\omega_0;\omega_1,...,\omega_N) \times\Phi(\mathbb{P}_{kj}\{\omega_0;\omega_1,...,\omega_N\}) = 0, \quad (9)$$

where $\mathbb{P}_{kj}$ interchanges the indices k, j with $k \le m, j \ge m+1$. Here Eq.(8) is for indistinguishability among the $m+1$ photons whereas Eq.(9) is for temporal distinguishability between the $m+1$ photons and the remaining $N-m$ photons. Eqs.(8, 9) can be equivalently written in time domain as

$$\begin{aligned}G(t_0;t_1,...,t_N) \\ = G(\mathbb{P}\{t_0;t_1,...,t_m\},t_{m+1},...,t_N)\end{aligned} \quad (10)$$

and

$$\int dt_0 dt_1 ... dt_N G^*(t_0;t_1,...,t_N) \times G(\mathbb{P}_{kj}\{t_0;t_1,...,t_N\}) = 0, \quad (11)$$

where the notations are the same as before and

$$G(t_0;t_1,...,t_N) \equiv \frac{1}{(2\pi)^{(N+1)/2}}\int d\omega_0 ... d\omega_N \Phi(\omega_0;\omega_1,...,\omega_N) e^{-i(\omega_0 t_0 + \omega_1 t_1 + ... + \omega_N t_N)}. \quad (12)$$

The $N+1$-photon coincidence rate of the $N+1$ detectors in Fig.1 is proportional to a time integral of the correlation function of [24]

$$\Gamma^{(N+1)}(t_0,t_1,...,t_N) = \langle\Phi_{N,1}|\hat{E}_1^{(o)\dagger}(t_N)...\hat{E}_1^{(o)\dagger}(t_1)\hat{E}_1^{(o)\dagger}(t_0)\hat{E}_1^{(o)}(t_0)\hat{E}_1^{(o)}(t_1)...\hat{E}_1^{(o)}(t_N)|\Phi_{N,1}\rangle. \quad (13)$$

where

$$\hat{E}_1^{(o)}(t) = \sqrt{T}\hat{E}_a(t) + \sqrt{R}\hat{E}_b(t) \qquad \text{with} \qquad \hat{E}_c(t) = (1/\sqrt{2\pi})\int d\omega \ \hat{c}(\omega) e^{-i\omega t} \qquad (c = a, b). \quad (14)$$

Let us first evaluate $\hat{E}_1^{(o)}(t_0)\hat{E}_1^{(o)}(t_1)...\hat{E}_1^{(o)}(t_N)|\Phi_{N,1}\rangle$, which has the form of

$$\hat{E}_1^{(o)}(t_0)\hat{E}_1^{(o)}(t_1)...\hat{E}_1^{(o)}(t_N)|\Phi_{N,1}\rangle = T^{N/2}R^{1/2}\sum_{k=0}^{N}\mathbb{P}_{0k}\{\hat{E}_b(t_0)\hat{E}_a(t_1)...\hat{E}_a(t_N)\}|\Phi_{N,1}\rangle \quad (15)$$

It is straightforward to show that for the state in Eq.(6), we have

$$\begin{aligned}\hat{E}_b(t_0)\hat{E}_a(t_1)...\hat{E}_a(t_N)|\Phi_{N,1}\rangle &= \frac{\mathcal{N}^{-1/2}}{(2\pi)^{(N+1)/2}}\int d\omega_0...d\omega_N \sum_{\mathbb{P}} \Phi(\omega_0;\mathbb{P}\{\omega_1,...,\omega_N\})e^{-i(\omega_0 t_0+...+\omega_N t_N)}|0\rangle \\ &= \mathcal{N}^{-1/2}\mathcal{G}(t_0;t_1,...,t_N)|0\rangle, \end{aligned} \tag{16}$$

where

$$\mathcal{G}(t_0;t_1,...,t_N) \equiv \sum_{\mathbb{P}} G(t_0;\mathbb{P}\{t_1,...,t_N\}). \tag{17}$$

The overall $(N+1)$-photon coincidence probability is then a time integral of the Γ-function in Eq.(13):

$$P_{N+1} = \int dt_0 dt_1...dt_N \Gamma^{(N+1)}(t_0,t_1,...,t_N). \tag{18}$$

With Eqs.(12, 15, 16, 17), we obtain

$$P_{N+1} = T^N R\mathcal{N}^{-1}\int dt_0dt_1...dt_N \sum_{k,j}\mathbb{P}_{0k}\{\mathcal{G}^*(t_0,t_1,...,t_N)\}\mathbb{P}_{0j}\{\mathcal{G}(t_0,t_1,...,t_N)\} = T^N R\bigg(\sum_{k=j}+\sum_{k\neq j}\bigg). \tag{19}$$

It is straightforward to find that the first term in Eq.(19) is

$$\sum_{k=j} = \mathcal{N}^{-1}(N+1)\int dt_0dt_1...dt_N\Big|\mathbb{P}_{0k}\{\mathcal{G}(t_0,t_1,...,t_N)\}\Big|^2 = \mathcal{N}^{-1}(N+1)\int dt_0dt_1...dt_N\Big|\mathcal{G}(t_0,t_1,...,t_N)\Big|^2, \tag{20}$$

where we switched the integral variables t_0 and t_k. Using Eq.(17) and the fact that $\sum_{\mathbb{P}}$ is over all permutations in $\{t_1,...,t_N\}$, Eq.(20) becomes

$$\sum_{k=j} = \mathcal{N}^{-1}(N+1)\int dt_0dt_1...dt_N N! G^*(t_0;t_1,...,t_N)\sum_{\mathbb{P}} G(t_0;\mathbb{P}\{t_1,...,t_N\}) = \mathcal{N}^{-1}(N+1)N!\mathcal{N} = (N+1)!, \tag{21}$$

where for an arbitrary permutation in $\mathcal{G}^*$, we also permute the variables of integral in the same way. Since $\mathcal{G}$ is unchanged under any such permutation, we obtain $N!$ identical terms in Eq.(21).

Next, let us consider the second term in Eq.(19). We evaluate one arbitrary term in the sum $(k\neq j)$:

$$\int dt_0...dt_N \sum_{\mathbb{P}} G^*(t_k,\mathbb{P}\{t_1,...,t_0,...,t_N\})\sum_{\mathbb{P}'} G(t_j,\mathbb{P}'\{t_1,...,t_0,...,t_N\}). \tag{22}$$

Because of the permutation properties in Eqs.(10, 11) and $t_k\neq t_j$, the only way to obtain a non-zero integral is for t_k in $\mathbb{P}'\{t_1,...,t_0,...,t_N\}$ to be permuted to the first m positions by $\mathbb{P}'$. Then we can use the permutation relation in Eq.(10) to interchange it with t_j so that for these $\mathbb{P}'$s, we have:

$$G(t_j,\mathbb{P}'\{t_1,...,t_0,...,t_N\}) = G(t_k,\mathbb{P}'\{t_1,...,t_0,...,t_N\}). \tag{23}$$

Permutation by $\mathbb{P}'$ to other positions for t_k cannot be interchanged with t_j and by the orthogonal relation in Eq.(11), the integral is zero. Since this is only about t_k in $\mathbb{P}'$, other $N-1$ time variables in $\mathbb{P}'$ are free to move. So, there will be $m(N-1)!$ permutation terms in the sum over $\mathbb{P}'$ that are nonzero and, as before in Eq.(21), they will all have the same time integral of $(N+1)\mathcal{N}$. Therefore, the second term in Eq.(19) is equal to

$$\sum_{k\neq j} = \mathcal{N}^{-1}\sum_{k\neq j} m(N-1)!\mathcal{N} = m(N+1)!. \tag{24}$$

Combining Eqs.(21, 24), we have

$$P_{N+1} = T^N R(1+m)(N+1)!. \tag{25}$$

The case of $m=0$ corresponds to the situation when the single photon of port b is completely distinguishable from all the N photons from port a, which gives no interference and sets a baseline for reference. Thus the enhancement factor for $m\neq 0$ is $m+1$, in agreement with the single mode analysis. Note that the enhancement factor does not depend on the transmissivity T but the detection probability P_{N+1} does. From Eq.(25), we find the maximum detection probability at $T = N/(N+1)$.

For the intermediate case when there are some partial indistinguishability among the photons, the wave function G does not satisfy Eqs.(10, 11). We will not obtain a simple close form as Eq.(25). However, for a special case when all the N photons from port a are indistinguishable from each other and they are only partially indistinguishable from the single photon input from port b, we have the permutation symmetry relation:

$$G(t_0; t_1, ..., t_N) = G(t_0; \mathbb{P}\{t_1, ..., t_N\}) \tag{26}$$

but no orthogonal relation similar to Eq.(11). Then Eq.(17) becomes

$$\mathcal{G}(t_0; t_1, ..., t_N) = N! G(t_0; t_1, ..., t_N). \tag{27}$$

With some manipulations, we find Eq.(19) becomes

$$\begin{aligned} P_{N+1} &= T^N R (N+1)! (1 + N\mathcal{V}_N) \\ &= P^{cl}_{N+1} (1 + N\mathcal{V}_N) \end{aligned} \tag{28}$$

with

$$\begin{aligned} \mathcal{V}_N &\equiv \frac{\int dt_0 dt_1 ... dt_N G^*(t_0; t_1, ..., t_N) G(t_1; t_0, ..., t_N)}{\int dt_0 dt_1 ... dt_N |G(t_0; t_1, ..., t_N)|^2} \\ &= \frac{\int d\omega_0 ... d\omega_N \Phi^*(\omega_0; \omega_1, ..., \omega_N) \Phi(\omega_1; \omega_0, ..., \omega_N)}{\int d\omega_0 ... d\omega_N |\Phi(\omega_0; \omega_1, ..., \omega_N)|^2}. \end{aligned} \tag{29}$$

Thus the enhancement factor is

$$\frac{P_{N+1}}{P^{cl}_{N+1}} = 1 + N\mathcal{V}_N. \tag{30}$$

Note that when $\mathcal{V}_N = 1$, we have the maximum enhancement factor of $N+1$, indicating complete indistinguishability whereas when $\mathcal{V}_N = 0$, there is no enhancement effect, due to complete distinguishability. So the quantity $\mathcal{V}_N$ gives the degree of distinguishability between the single photon in port b and the N photons in port a.

The more general case is when there are m indistinguishable photons among the N photons in port a and the m photons have partial indistinguishability from the single photon in port b. To simplify the case, we further assume that the other $N - m$ photons from port a are completely distinguishable from the above $m + 1$ photons. For this case, we can show similar to Eq.(30) that the enhancement factor is

$$\frac{P_{N+1}}{P^{cl}_{N+1}} = 1 + m\mathcal{V}_m, \tag{31}$$

where $\mathcal{V}_m$ is defined in Eq.(29) but with the wave function G satisfying the permutation symmetry relation in Eq.(10) in stead of Eq.(26).

Now the experimental procedure to measure the distinguishability of the N-photon state is depicted in Fig.2, where we scan the relative delay of the single photon in port b with respect to the N-photon state in port a. Whenever the single photon scans through m indistinguishable photons, the $N + 1$ coincidence count shows a bump of size m relative to the baseline. In this way, we can characterize the temporal distinguishability of the N-photon state. The width of the bump is determined by the function $\mathcal{V}_m$.

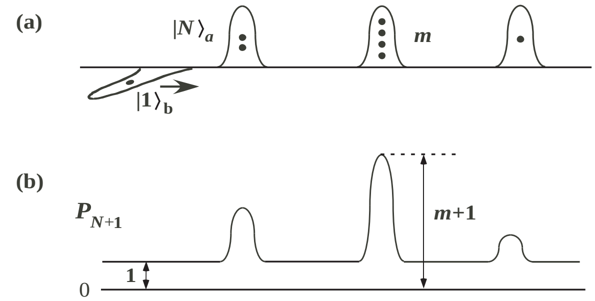

FIG. 2: (a) A temporal distribution with well-separated groups of N photons and (b) the corresponding normalized P_{N+1} as the position of the single photon is scanned.

IV. MORE GENERAL CASE OF $|N_a, M_b\rangle$

For the more general input state of $|N_a, M_b\rangle$, we can use the analysis similar to Eqs.(4, 5). First, if the $N+M$ photons are indistinguishable, then from the quantum theory of a lossless beam splitter [23], we may find the probability of finding all $N + M$ photons in one output side of the beam splitter as

$$P_{N+M} = T^N R^M (N + M)!/N!M!. \tag{32}$$

But when the incoming N photons are distinguishable from the M photons, the $N + M$ photons act like classical particles and follow the probability law. The corresponding classical probability is then

$$P^{cl}_{N+M} = T^N R^M. \tag{33}$$

So the enhancement factor due to quantum interference is

$$\frac{P_{N+M}}{P^{cl}_{N+M}} = \frac{(N + M)!}{N!M!}. \tag{34}$$

A special case is for $N = M = 2$, which gives the ratio of 6. This is the photon pair bunching effect experimentally demonstrated by Ou et al [10].

The photon bunching enhancement factor in Eq.(34) is for all the photons involved to be indistinguishable. When some of the photons are distinguishable, the enhancement factor will decrease. The most general scenario is that some of the N photons at input port a are indistinguishable from some of the M photons at side b. Let's break the N photons and the M photons into $k+1$ groups, respectively, namely, $N = n_1 + ... + n_k + n_{k+1}$ and $M = m_1 + ... + m_k + m_{k+1}$. In these groups, the n_i photons are indistinguishable from m_i photons with $i = 1, 2, ..., k$ and $\{n_i, m_i\}$ group of photons are distinguishable from $\{n_j, m_j\}$ group of photons with $i \neq j$. Furthermore, the n_{k+1} photons are distinguishable from m_{k+1} photons. Such a situation is depicted in Fig.3.

In analogy to the case of stimulated emission described in Eq.(5), we may write the input state to the beam split-

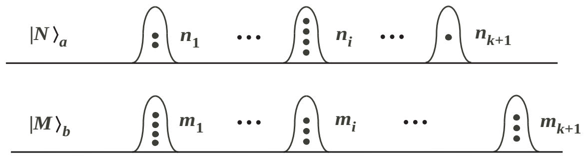

FIG. 3: Temporal distributions for photons from input sides a and b, respectively.

TABLE I: Enhancement factor for 2 a-photons and 2 b-photons input

$2a2b$	$2a1b + 1b$	$1ab{+}1ab$	$1ab{+}a + b$
6	3	4	2

ter as

$$|\Phi\rangle_{in} = |n_{k+1}\rangle_a^{(k+1)} \otimes |m_{k+1}\rangle_b^{(k+1)} \prod_{i=1}^{k} \otimes (|n_i\rangle_a^{(i)} |m_i\rangle_b^{(i)}), \tag{35}$$

where we use $\otimes$ and superscript "(i)" to separate and label the states of distinguishable photons.

Since $|n_i\rangle_a|m_i\rangle_b$ is the same state as $|N_a, M_b\rangle$ that gives rise to the enhancement factor in Eq.(34), it will contribute a factor of $(n_i + m_i)!/n_i!m_i!$ to the overall enhancement factor, which is then

$$\frac{P_{\{n_i,m_i\}}}{P^{cl}_{N+M}} = \prod_{i=1}^{k} \frac{(n_i + m_i)!}{n_i!m_i!}. \tag{36}$$

Note that since $|n_{k+1}\rangle_a$ and $|m_{k+1}\rangle_b$ are distinguishable states, they have no contribution to the enhancement factor.

In Tables I-III, we list the enhancement factors for various scenarios of the input states $|2_a, 2_b\rangle, |3_a, 2_b\rangle, |3_a, 3_b\rangle$, respectively. These tables are in contrast with the visibility tables in Ref.[14].

V. SUMMARY AND DISCUSSION

In this paper, we discussed a generalized photon bunching effect which may involve arbitrary number of photons. This bunching effect is a result of constructive multi-photon interference and is responsible for stimulated emission of an excited atom. Furthermore, we find

TABLE II: Enhancement factor for 3 a-photons and 2 b-photons input

$3a2b$	$2a2b$ $+a$	$3a1b$ $+b$	$2a1b$ $+ab$	$1a2b$ $+2a$	$2a1b$ $+a+b$	$ab{+}a$ $+ab$	$ab{+}a$ $+a+b$
10	6	4	6	3	3	4	2

the bunching effect can be used to characterize temporal distinguishability of photons: various scenarios of photon temporal distribution give different enhancement factors.

From Eq.(36), we find that the larger the photon number, the larger the enhancement factor and it is largest for the case of $N = M$. The largeness of the enhancement factor will make the measurement process easier. Although the enhancement factor in Eq.(36) does not depend on T, R, the actual value of the probability in Eq.(32) does and it is maximum when $T = N/(N + M)$. From Tables I-III in comparison with the tables in Ref.[14], we find that the values here are more spreading out and this also makes it easier to tell them apart experimentally.

In this paper, we only considered some extreme cases, i.e., the photons are either completely indistinguishable, as described in Eq.(8), or completely distinguishable, as in Eq.(9). The intermediate case is very complicated and it is hard to derive the enhancement factor in a closed form. For some simple cases, however, we are able to do it. For example, Eq.(4.20) in Ref.[11] provides a formula for the enhancement factor for the input state of $|2_a, 2_b\rangle$ from parametric down-conversion. It was found that the enhancement factor depends on the quantity $\mathcal{E}/\mathcal{A}$, which characterizes the degree of distinguishability between different pairs of photons in parametric down-conversion.

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TABLE III: Enhancement factor for 3 a-photons and 3 b-photons input

$3a3b$	$3a2b$ $+b$	$3a1b$ $+2b$	$2a2b$ $+ab$	$2a2b$ $+a+b$	$2a1b$ $+1a2b$	$2a1b$ $+1ab+b$	$2a1b$ $+a+b+b$	$ab{\times}3$	$ab{\times}2$ $+a+b$	$ab+b$ $+a+a+b$
20	10	4	12	6	9	6	3	8	4	2

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